Higher structures appear increasingly in a variety of contexts: mathematical physics, algebraic topology, knot theory, representation theory, with the aim of providing finer and finer invariants for the objects under study (spaces groups, etc.). They arise naturally when weakening structures with a binary operation that is associative (typically, a category) to ones where associativity holds up to homotopy (typically a bicategory), and then when there are homotopies between homotopies, etc. This thesis project will take place in one of the formalisms that have been proposed to define such higher structures: the opetopes and the opetopic sets of Baez and Dolan (further studied by Cheng, Leinster, and others).

The hosting team (PPS team of IRIF, UFR d’Informatique, Université Paris Diderot) has a leading expertise in higher-dimensional rewriting. Rewriting theory is historically concerned with words and terms (and higher-order terms, like in the $\lambda$-calculus). An extension of the coverage of rewriting techniques to deal with higher structures is a timely subject, given its applications to effective and mechanizable methods for computing various mathematical invariants, and to the proofs of some important algebraic properties, like koszulity (a corner stone in algebraic operad theory).

The main goal of the thesis will be to study opetopes from the point of view of rewriting theory. They seem to be sufficiently general to encompass most of the abstract concepts that are now used by the communities working with higher structures, in particular, operads and their variants. At the same time, they seem to be sufficiently restricted with respect to the fuller generality offered by polygraphs (also known as computads), which so far have been the support for the research in higher-dimensional rewriting (works of Lafont, Guiraud, Malbos, Mimram, among others, building on pioneering independent works of Burroni and Street), the state-of-the-art being somehow blocked by a number of difficulties that can be treated in examples but seem to stand in the way for a general theory.

The thesis work will develop on one hand general results and techniques for “opetopic rewriting”, such as criteria for convergence, i.e. confluence and termination.

The key difference between opetopic sets and polygraphs is that the former are a theory for (higher-dimensional) operations with many inputs and one output, while polygraphs allow multiple outputs. This extra generality causes problems in the effectivity of checking the local confluence of an oriented presentation. We believe that these problems will vanish by restricting our attention to the opetopic setting.

On the other hand, the thesis project will hopefully lead to interesting forefront applications. We are targeting in particular further generalisations of Koszul duality.

Historically, Koszul duality was first developed for associative algebras, and then transferred by Ginzburg and Kapranov to the level of operads, which encode algebraic structures among which associative algebras, but also Lie algebras, Poisson algebras, etc. The works of Berger, Hoffbeck, Dotsenko, among others, have shown a strong link between koszulity and convergence, namely: quadratic operads with a convergent presentation (or, equivalently, for which a
Gröbner basis can be found) are koszul. The importance of koszulity in applications lies in the fact that being koszul entails having associated easy-to-compute homology.

Given all that, the next natural step is to investigate the structures that encode operads and their variants (such as cyclic operads, wheeled operads, permutads, etc.), and so on. We believe that this can be done within the opetopic framework. Whence the (ambitious) objective of lifting koszul duality to the opetopic level. Since koszulity is intimately linked with rewriting, this second objective is in tune with the first.

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**References**


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